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Abstract — In this paper we propose a method to obtain performance characteristics of Single Phase AC Voltage Controller feeding R and R-L load. We also analyze the Harmonics of AC voltage controller feeding R and RL load. We apply numerical techniques viz. Simpson’s 1/3rd rule, Euler’s Rule and Newton-Raphson method in order to test the performance characteristics. The performance characteristics such as output voltage, output current, power factor are evaluated with respect to the variation in firing angle α. This paper elucidates some basic methods and techniques for programming simulations of differential and Integral Equations. The fundamental concepts of numerical methods will be introduced along with practical implementation programs. We demonstrate the results of the work carried out and observe that the results obtained by applying numerical techniques are in line with the experimental and simulation results.

Index Terms — Harmonic analysis, Firing angle, Numerical Techniques

I. INTRODUCTION

AC to AC voltage converters operates on the AC mains essentially to regulate the output voltage. Portions of the supply sinusoid appear at the load while the semiconductor switches block the remaining portions. The AC power can be regulated by two methods: first one is on-off control and second one is phase angle control [4] and [5]. In the method of phase control, switches connected the load to the AC source for a portion of each cycle of input voltage. The main unpalatable point in AC voltage controller is the introduction of harmonics in power quantities waveform at reduced output voltage levels. The AC voltage regulator is used as one of the power electronic systems to control output. AC voltage for power ranges from a few watts (as in light dimmers) up to fractions of megawatts (as in starting systems for large induction motors).[4]

AC-AC phase-angle control with Thyristors technology has been widely used. However, these techniques has many drawbacks such as 1. Retardation of the firing angle causes a lagging power factor at the input side. 2. Plentiful low order harmonics in both of supply voltages and currents. 3. Discontinuity of power flow to the load.

Authors in [1] have discussed about the performance characteristics of thyristorized inverse parallel controller (IPC) under a single-phase induction motor load. They have used MATLAB 7.8 to implement their work . Author in [2] has discussed about harmonic distortion at the output voltage of AC voltage controller circuit using OrCAD Lite 16.6. However C Programming language offers better flexibility and structuring when compared to other softwares.


We Discuss the Methodology in Section II, Harmonic analysis in Section III, Results and Discussions in Section IV and give conclusions in V.

II. METHODOLOGY

The proposed work as shown in the Fig. 1.0 was carried out on R and R-L Load. Different numerical techniques viz. were used in order to evaluate the performance characteristics of Single Phase AC Voltage Controller .The different performance characteristics such as output voltage, output current and power factor were evaluated with varying firing angle α. Harmonic Analysis has been carried out and harmonic content for the output voltage signal was estimated.

![Fig 1.0 Block Diagram representing Methodology Analysis of AC Voltage Controller feeding R Load](Image)
Fig 2.0 AC Voltage Controller Feeding R Load.

Let the source voltage be \( V_s(\omega t) = V_m \sin \omega t \) as shown in Fig 2.0 feeding a resistive load. The RMS load voltage is determined by taking advantage of positive and negative symmetry of the voltage waveform, necessitating evaluation of only a half period of the waveform.

The output Voltage (RMS) is given by

\[
V_o = \sqrt{\frac{1}{T} \int_0^T (V_m \sin \omega t)^2 \, dt (\omega t)}
\]  

(1)

As the load is resistive in nature the output Current (RMS) is given by

\[
l_o = \frac{V_o}{R}
\]  

(2)

The Load power factor is evaluated as

\[
p_f = \frac{1 - \frac{a}{\pi} + \frac{\sin 2a}{2\pi}}
\]  

(3)

To evaluate the Integral equation (1) we make use of Simpson’s 1/3rd rule.

**SIMPSONS’ 1/3rd RULE**

Simpson’s Rule Formula is used to calculate the integral value of any function. It calculates the value of the area under any curve over a given interval by dividing the area into equal parts. [3]

\[
y = \frac{1}{3} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]
\]

Where step length, \( h = \frac{b-a}{n} \)

a is lower limit, \( b \) is upper limit, \( n \) is number of iterations.

Analysis of AC Voltage Controller Feeding R-L Load.

Fig 3.0 AC Voltage Controller Feeding R-L Load

Kirchhoff’s voltage law for the Fig 3.0 is expressed as

\[
V_m \sin \omega t = R i_o(t) + L \frac{di_o(t)}{dt}
\]  

(4)

The solution for current in this equation is:

\[
l_o = \frac{V_m}{Z} [\sin(\omega t - \alpha) - \sin(\omega t - \alpha) e^{\frac{\omega t - \beta}{\omega \tau}}] \quad (\alpha < = \omega t < = \beta)
\]  

(5)

The extinction angle \( \beta \) is the angle at which the current returns to zero, when \( \omega t = \beta \).

\[
l_o(\beta) = \frac{V_m}{Z} [\sin(\beta - \alpha) - \sin(\alpha - \beta) e^{\frac{\omega t - \beta}{\omega \tau}}] = 0
\]  

(6)

which must be solved numerically for \( \beta \).

In the interval between \( \pi \) and \( \beta \) when the source voltage is negative and the load current is still positive, \( S_2 \) cannot be turned on because it is not forward biased. The gate signal to \( S_2 \) must be delayed at least until the current in \( S_1 \) reaches zero, at \( \omega t = \beta \).

The conduction angle is:

\[
\delta = \beta - \alpha
\]  

(7)

To solve the transcendental equation (6) we make use of Newton Raphson method and Euler’s Rule to validate the same.

**NEWTON RAPHSON METHOD**

The Newton-Raphson method, or Newton Method, is a powerful technique for solving differential equations numerically. Like so much of the differential calculus, it is based on the simple idea of Linear approximation. [3]

\[
x_{k+1} = x_k - \frac{f(x)}{f'}(x)
\]  

(8)

Here the function is :

\[
f(x) = l_o = \frac{V_m}{Z} [\sin(\omega t - \alpha) - \sin(\alpha - \beta) e^{\frac{\omega t - \beta}{\omega \tau}}]
\]  

(9)

\[
f'(x) = \frac{dV(t)}{dt} = \frac{V_m}{Z} [\cos(\omega t - \alpha) + \frac{\sin(\alpha - \beta)}{\omega t} e^{\frac{\omega t - \beta}{\omega \tau}}]
\]  

(10)

**EULER’S RULE**

Euler’s Rule is numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations.

\[
V_m \sin \omega t = R i_o(t) + L \frac{di_o(t)}{dt}
\]  

(11)

\[
f(x) = \frac{di_o(t)}{dt} = \frac{V_m \sin \omega t - i_o(t) \tau}{L}
\]  

(12)

\[
l_{(n+1)} = l_n + f(x) * h
\]  

(13)

**III. HARMONIC ANALYSIS**

Due to non-linear behavior of the controller and the load, the output voltage is non-sinusoidal i.e. distorted. This distortion in the voltage output waveform is called Voltage Harmonic Distortion which is analyzed using Fourier analysis of the explicit expressions [1] for the instantaneous output voltage over a period of the voltage waveform as follows:

\[
V(\omega t) = \sum_{n=2}^{\infty} A_n \sin n\omega t + B_n \cos n\omega t
\]  

(14)
Where $A_n$ and $B_n$ are expressed as:

$$A_n = \frac{1}{T} \int_{0}^{T} V(\omega t) \cos n\omega t \, d(\omega t)$$

$$B_n = \frac{1}{T} \int_{0}^{T} V(\omega t) \sin n\omega t \, d(\omega t)$$

$T$ is the period.

$A_n$ and $B_n$ are the Maximum value of the sine and cosine components of the harmonics of order $n$; present in the output voltage waveform respectively. If the amplitude (maximum value) and the phase angle of the $n^{th}$ harmonic component are $C_n$ and $\theta_n$ respectively which are define as:

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\theta_n = \tan^{-1} \frac{B_n}{A_n}$$

Only the odd harmonics are present in output voltage because of the half-wave symmetry.[1]

Harmonic components derived for different intervals of operation are:

$$A_n = \frac{1}{n+1} (\cos(n+1)\alpha - \cos(n+1)\phi) - \frac{1}{n-1} (\cos(n-1)\alpha - \cos(n-1)\phi)$$

$$B_n = \frac{1}{n+1} (\sin(n+1)\alpha - \sin(n+1)\phi) - \frac{1}{n-1} (\sin(n-1)\alpha - \sin(n-1)\phi)$$

IV. RESULTS AND DISCUSSIONS

We demonstrate the performance characteristics of Single Phase AC Voltage Controller proposed for both R and R-L load using proposed numerical techniques and GNU Plot.

For AC input voltage of 230sin100t feeding a resistance of 10Ω, Load voltage is plotted against varying firing angle. Note that for $\alpha = 0$ in Fig 2.1, the load voltage is a sinusoid that has the same RMS value as the source. [4]

Load Current is plotted against varying firing angle. In Fig 2.2 we observe that as the value of firing angle $\alpha$ increases, the load RMS current decreases.

In Fig 2.3 note that power factor = 1 for $\alpha=0$, which is same as for uncontrolled resistive load and the power factor for $\alpha$ greater than zero is less than 1. [4]

As evidently seen from the Fig 2.4, the harmonic content is maximum for the fundamental frequency $n$, and later on the content decreases with increase in integral multiple of order $n$.

For AC input voltage of 120V(rms) feeding a resistance of 20Ω and Inductance of 50mH and frequency of 60Hz, Load voltage is plotted against varying firing angle.
The SCR is being fired at $\alpha=90^\circ$. As seen in Fig 3.1 the value of $\alpha$ goes on increasing, the load value of RMS voltage goes on decreasing.[4]

![Fig 3.2 Firing Angle Vs Load RMS Current](image)

The value of $\beta$ obtained by using Euler’s formula is 221.96 which is evident in Fig 3.4. The value of $\beta$ obtained from Newton-Raphson method is 221.44°. The value of $\beta$ obtained from Newton-Raphson method is verified using Euler’s Rule.

![Fig 3.3 Firing angle Vs Power factor](image)

As evidently seen from the Fig 3.5, the harmonic content is maximum for the fundamental frequency $n$, and later on the content decreases with increase in integral multiple of order $n$.

![Fig 3.4 R-L Load Current Waveform](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experimental Result</th>
<th>Simulation Result</th>
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</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>3.83</td>
<td>3.89</td>
</tr>
<tr>
<td>Load Voltage(RMS)</td>
<td>91.28</td>
<td>91.03</td>
</tr>
<tr>
<td>Load</td>
<td>2.71</td>
<td>2.71</td>
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</tbody>
</table>

Table 1.0 Comparison of Results

**CONCLUSION AND FUTURE SCOPE**

In this paper we have made an attempt to obtain performance characteristics of Single Phase AC Voltage controller with R and R-L Load using Numerical techniques, C language and Data Structures. We have tested performance characteristics namely output voltage, output current and load power factor with variation of Firing angle $\alpha$ using numerical techniques such as Simpson’s 1/3rd rule, Newton-Raphson Method and Euler’s method. Harmonics of Single Phase AC Voltage controller are also analyzed using Fourier Series.

It is observed that the results obtained from numerical techniques are in line with experimental values.

Performance Analysis of Single Phase AC Voltage Controller is analyzed, this can be taken further in analyzing Three Phase AC Voltage Controller and an attempt can be made to reduce the lower order harmonics.

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**REFERENCES**


