Analysis of the Torque-Speed Characteristics of a Chopper Fed DC Series Motor

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Abstract — **The paper describes simulation results of a DC series motor, considering its non-linear magnetization characteristics. DC series motor has high starting torque and a characteristic non-linear magnetization curve. The advantage of using DC series motor is the possibility of changing the direction of rotation by reversing the polarity of the Input Voltage. This paper deals with analysis of Torque-Speed characteristics of a DC series motor connected to a chopper, using the numerical techniques. Code to obtain unknown armature voltage values using the magnetization characteristics of the motor and to calculate various motor parameters like torque, speed etc. is presented. Results are plotted using the GNU plot.**

Index Terms — **Non-linear Characteristic magnetization curve, Torque-speed characteristics, Numerical techniques, GNU Plot.**

I. Introduction

Self-excited series dc motors are commercially used in large scale. A high torque is obtained at low speed and a low torque is obtained at high speed-an important characteristic known at the series motor characteristic. Series motors are used to drive loads at various speeds and variable torques. Speed of Self excited series dc motors can be controlled by various methods. However chopper circuits can be used for this purpose. DC Chopper can be effective in controlling the speed of the motor. However in various applications, calculation of load torque required to drive the load at the output side is computed. Hence, Torque-speed characteristics are essential.
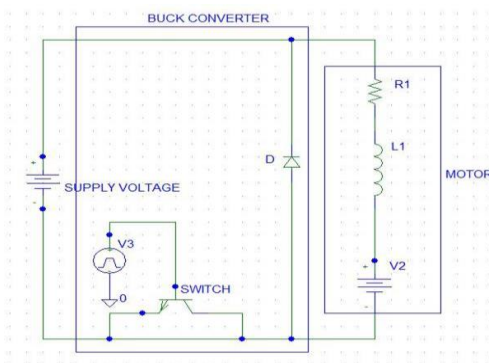
II. Methodology

A. DC-Drive system

Figure 1. Pspice model of the Drive circuit.

Circuit parameters:-
V - Supply voltage
R_a – Armature resistance
R_{se} -Series resistance
I_a – Armature current.

By applying Kirchhoff's voltage law[KVL]:
1. During ON-Period of the switch:

$$V = E + (R_A + R_{SE})I_A + L_A \times dI_A/dt \quad , 0<t<t_{on} \qquad (1)$$

2. During OFF-Period of the switch:

$$V = 0 = E + (R_A + R_{SE})I_A + L_A \times \frac{dI_A}{dt} \quad , t_{on} < t < T \qquad (2)$$

3. Duty ratio is given by:

$$\delta = \frac{on-time}{time\ period} = \frac{t_{on}}{T} \qquad (3)$$

4. Equations obtained from figure 1:

$$V_a = \left(\frac{1}{T}\right) \times \int_0^{t_{on}} V.dt = \delta V \qquad (4)$$

$$I_a = \frac{\delta V - E}{R_a + R_{se}} \qquad (5)$$

5. Expression relating Torque, speed of the motor and duty cycle of the chopper is given by:

$$w_m = \frac{\delta V}{K} - \frac{(R_a+R_{se})T}{K^2} = \left(\frac{V}{K\emptyset}\right) - \left(\frac{I_aR}{K\emptyset}\right) \quad (\text{since } I_a \propto T) \qquad (6)$$

6. Nature of Torque-Speed characteristics of the drive circuit for different values of dutyratio :
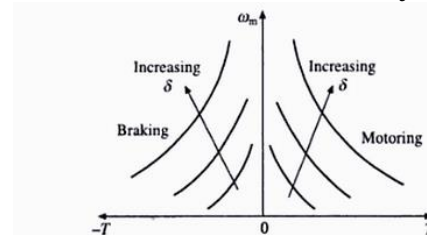

Figure 2. Torque-Speed characteristics with duty ratio.

B. Numerical methods

1. Newton's Interpolation method:

The process of finding the y value for any intermediate value of x in the interval is called interpolation and the process of finding the value of y for the value of x outside the interval is called extrapolation.

1.1 Newton's forward Interpolation method:

Table 1. Forward differences table

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_1	y_1			

		Δy_1		
x_2	y_2		$\Delta^2 y_1$	
		Δy_2		$\Delta^3 y_1$
x_3	y_3		$\Delta^2 y_2$	
		Δy_3		
x_4	y_4			

Nth order forward differences:

$$\Delta y_1 = y_2 - y_1$$

$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$

$$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$$

Newton's Forward Interpolation Formula:

$$Y = y_1 + p\Delta y_1 + \frac{p(p-1)\Delta^2 y_1}{2!} + \frac{p(p-1)(p-2)\Delta^3 y_1}{3!} \qquad (7)$$

Where p=x-x_1/h .

h is the finite difference between x terms.

1.2 Newton's backward Interpolation method:

Table 2. Backward differences table

X	Y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_1	y_1			
		∇y_1		
x_2	y_2		$\nabla^2 y_1$	
		∇y_2		$\nabla^3 y_1$
x_3	y_3		$\nabla^2 y_2$	
		∇y_3		
x_4	y_4			

Nth order backward differences:

$$\nabla y_1 = y_1 - y_2$$

$$\nabla^2 y_1 = \nabla y_1 - \nabla y_2$$

$$\nabla^3 y_1 = \nabla^2 y_1 - \nabla^2 y_2$$

Newton's Backward Interpolation Formula:

$$Y = y_1 + p\nabla y_1 + \frac{p(p+1)\nabla^2 y_1}{2!} + \frac{p(p+1)(p+2)\nabla^3 y_1}{3!} \qquad (8)$$

Where p=x-x_1/h .

h is the finite difference between x terms.

1.3 Newton's divided difference method:

This can be used for unequally spaced x variables;

We have (x_i, y_i) where i=0, 1, 2 ...n;

Newton's Divided Differences:

$$(x_0, x_1) = y_2 - y_0/x_1 - x_0$$

$$(x_1, x_2) = y_2 - y_1/x_2 - x_1$$

$$(x_2, x_3) = y_3 - y_2/x_3 - x_2$$

$$(x_0, x_1, x_2) = (x_0, x_1) - (x_1, x_2)/x_0 - x_2$$

$$(x_1, x_2, x_3) = (x_1, x_2) - (x_2, x_3)/ (x_1 - x_3)$$

$$(x_0, x_1, x_2, x_3) = (x_0, x_1, x_2) - (x_0, x_1, x_2)/(x_0 - x_3)$$

Newton's Divided Differences Formula:

$$Y = y_1 + (x - x_0)(x_0, x_1) + (x - x_0)(x - x_1)$$
$$(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2)(x_0, x_1, x_2, x_3) \qquad (9)$$

C. Methodology

1. D.C motor is connected to the chopper and supply voltage is given to the chopper. Values from the Magnetization curve of the motor are tabulated.

2. Codes are written using c programming and data structures for the numerical methods and also to compute values of load torque and speed of the drive.

3. Magnetization Characteristics of the motor is plotted and unknown armature voltage values are obtained using the code of numerical method.

4. Magnetization curve is replotted with the obtained values from the code and checked for the error by plotting the magnetization.

Curve and the interpolation plot of intermediate values.

5. Results are compared from the codes of two numerical methods and are plotted.

6. Using the results from the numerical method, torque and speed values are tabulated for different duty ratios of the chopper from the code.

7. Torque-speed values are plotted using GNU plot for different duty ratios of the chopper.

8. Analyze Torque-speed characteristics of the motor with different duty ratio values of the chopper and other plots indicating motor performance.

C.1 Magnetization Characteristics:

The magnetization curve of a chopper connected dc series motor is of great importance because it represents the saturation level in the magnetic system of the dc motor for various values of the excitation mmf at a particular speed. The induced voltage in the armature winding is directly proportional to ux time's speed. Hence, Magnetization curve can also be expressed in terms of armature induced voltage and field current.

C.2 Applying numerical method and Magnetization curve analysis:

Magnetization curve is obtained experimentally by rotating the dc motor at various values of duty cycles of the converter; and measuring the open circuit armature terminal voltage as the current in the field winding is changed. But we can experimentally calculate the values for a single range. Hence to find out voltage values at variable values of current in the range; we can use numerical techniques like interpolation methods.

D. Pseudo Code

The pseudo code for analysis of Numerical Methods using magnetization curve values and for finding the torque-speed values:

```
Include header file
main ()
{
Read the total number of magnetization characteristics values at given speed, input voltage and the total resistance of the motor;
for (readings of magnetization values)
{
Read the armature current (Iₐ) and armature voltage (Eₐ) for series motor in multidimensional array;
}
for (readings of magnetization values)
{
Find the forward and backward differences;
Read the differences in the corresponding file;
```

}
do
{
Read the I_a value from the range for which E_a value has to found out;
if (the entered x value is less than the central value of x terms)
{
Apply the forward interpolation to find E_a for the intermediate reading of I_a at known speed;
Enter the duty ratio;
Find the E_a value for the unknown speed using the formula:

E_a=Input voltage*duty ratio-(I_a)*(total resistance);

Unknown speed=known speed*(E_a for unknown speed/E_a for known speed);
Unknown speed in rad/sec=2*pi*unknown speed;
Torque= E_a for unknown speed*I_a/unknown speed in rad per sec;
}
Read the Torque, speed, armature voltage and armature current Values in the corresponding file;
}
else
{
Apply the backward interpolation to find E_a values for the intermediate reading I_a for known speed;
Repeat the above steps to compute the values;
}
}
while(check the condition for true or false)
If true, enter more readings and enter false to exit.
}

III. IMPLEMENTATION

Consider a standard example to verify interpolation code:
Given: The following magnetization characteristics at 600 RPM.

Field current(A)	Armature voltage(V)
10	64
20	118
30	150
40	170
50	184
60	194
70	202
80	210

Q)Find the Armature voltage values for I_a values:25A,55A,75A .
Solution:
Armature voltage values are found out by applying the code for numerical methods ; Newton's forward and backward interpolation or Newton's divided difference method. Output values obtained are:
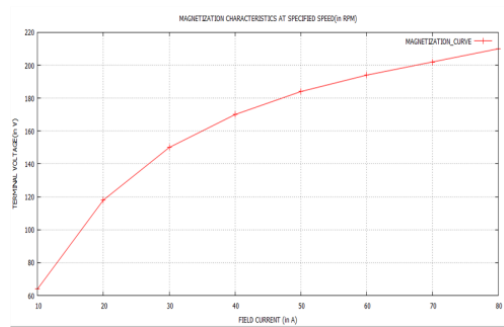25A -> 136.05 V
55A -> 189.41 V
75A -> 206.08 V

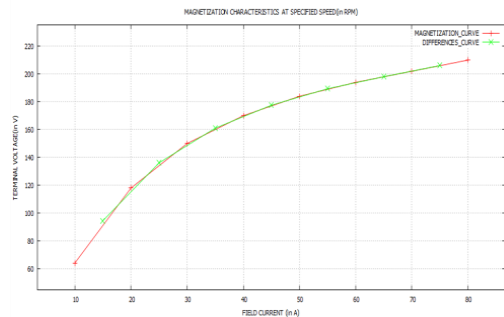Figure 3. Magnetization characteristics

Figure 4. Error identification curve

A. Realization of Torque-Speed characteristics

Extending the above standard problem :
A 220V, 70A dc series motor has combined resistance of armature and field is 0.12 running at no load with the field windings connected to the supply voltage of 250V.Magnetization characteristics table at 600 rpm is considered. Find the speed and torque parameters of the motor at duty ratios 0.4, 0.6, 0.8 for currents:
15A,25A, 35A, 45A, 55A, 65A, 75A.
solution:
Given :
Input to chopper (V_t)=220V
Total internal resistance (R_{total})=0.12
Speed (N_1) = 600rpm.
1) For duty ratio: 0.6
Case (a): Take I_a=15A
$V_{a1} = \partial*V_t = 220*0.6=132$ V
By using the KVL equation,

$$V_{a1} = E_{a1} + I_{a1}*R_{total} \quad (10)$$

Solving the equation for E_{a1},we get
$E_{a1} = V_{a1}-I_{a1}*R_{total}$
$E_{a1} = (132-15 *0.12)$ V
$E_{a1} =130.2$ V
The above calculated armature voltage value is for unknown speed of the motor.
From the magnetization characteristics values, we cannot find the E_a value for I_a =15A. So we apply the code of Newton's interpolation techniques to get E_a value for the given speed of the magnetization characteristics of the motor.
Therefore

I_a =15A -> E_a=94.03V
E_a =94.03->N=600 RPM
E_{a1} =130.2-> N RPM.

we obtain ,
Speed 'N' of motor(in RPM) =(E_1/E_a*N_1)=827.11 RPM
Speed (in RPS)=(2*N(in rpm)*pi/60)
we know that,
Mechanical power=Electrical power
$T*w=E_a*I_a$ \quad (11)

Therefore,

$$\text{Torque 'T'} = (E_a * I_a)/w = 22.54 \text{Nm}.$$

A. Results obtained for multiple values and different duty ratio

Table 1. Duty ratio 0.4

Field current(A)	Armature Voltage(V)	Speed (RPM)	Speed (RPS)	Torque (Nm)
15	94.44	827.11	86.62	22.54
25	136.65	568.86	59.57	54.12
35	166.011	476.23	49.87	89.67
45	177.55	427.8	44.805	127.15
55	189.41	397.22	41.602	165.78
65	198.06	376.24	39.405	204.87
75	206.08	358.10	37.5	245.96

Table 2. Duty ratio 0.6

Field current(A)	Armature Voltage(V)	Speed (RPM)	Speed (RPS)	Torque (Nm)
15	94.44	1106.6	115.91	22.54
25	136.65	762.90	79.90	54.12
35	166.01	640.20	67.05	89.67
45	177.55	576.48	60.37	127.1
55	189.41	536.6	56.2	165.7
65	198.06	509.53	53.36	204.8
75	206.08	486.2	50.92	245.9

Table 3. Duty ratio 0.8

Field current(A)	Armature Voltage(V)	Speed (RPM)	Speed RPS	Torque (Nm)
15	94.44	547.59	57.35	22.54
25	136.65	374.83	39.25	54.12
35	166.01	312.25	32.7	89.67
45	177.55	279.11	29.23	127.15
55	189.41	257.64	27.05	165.78
65	198.06	242.95	25.44	204.87
75	206.08	230.01	24.08	245.9

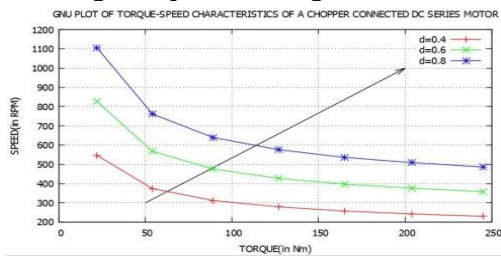These readings are plotted using GNU PLOT :

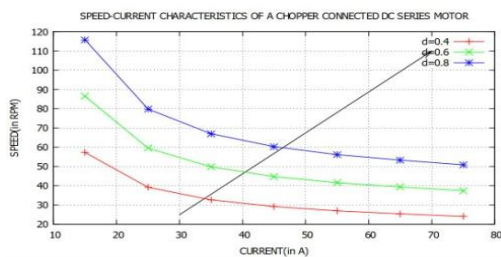Figure 5: Torque-speed characteristics

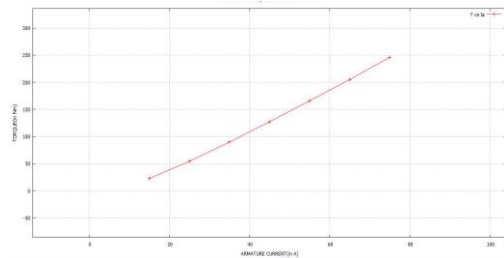Figure 6: Speed-Current characteristics

Figure 7: Torque-current characteristics

CONCLUSION

Nature of *TORQUE-SPEED CHARACTERISTICS* shifts upwards on increasing duty ratio of the Chopper which is connected to the dc series Motor is verified by coding and GNU PLOTS.

This analysis is very helpful in computing required torque and speed to drive the load connected to the dc series motor with chopper .This drive system is found in locomotives, heavy weight lifting equipment, machinery, etc.

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