Selective Harmonic Elimination using Graphical Analysis Method in a 5-Level Inverter fed from Equal and Non-Equal DC Sources

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Abstract One of the important requirements of multilevel inverters is to eliminate desired harmonics and improve the quality of output. This requirement is best met by employing suitable modulation strategy. Most strategies, involve computationally complex algorithms to be executed by the microcomputer. This leaves the microcomputer with little room for performing other critical system tasks. Selective Harmonic Elimination (SHE) techniques, in specific, the graphical analysis method can provide this simplicity by relieving the microcomputer from the computational stress. In this reported work, graphical analysis method is used for reduction of targeted harmonics in a 5-level cascaded inverter. The results obtained shows that the targeted harmonics are eliminated completely for inverters with both equal and non-equal dc sources.

Index Terms— Cascaded multilevel inverter, Graphical Analysis Method, Selective Harmonic Elimination.

I. INTRODUCTION

Multilevel inverters just like conventional inverter types are used to provide regulated ac power from dc sources. The key difference between the two lies in the quality of output produced by them. Multilevel inverters have the inherent capability to produce stepped waveforms resembling more closely to a sinusoidal waveform. When compared to conventional inverters, multilevel inverters have a number of advantages such as capability to produce reduced harmonic distortion, flexibility to be operated at fundamental or high switching frequencies, draws current from the sources with less distortion, reduced switching stress, lesser electromagnetic interferences, ability to produce smaller common mode voltage [1]. Because of these advantages they are widely getting accepted for use in industrial applications. The structure of these multilevel inverters makes them well suited for high power, medium voltage applications [2].

The different arrangements of self-commutated controlled power switches like MOSFETs, IGBTs in these multilevel inverters has given rise to numerous topologies, but in general they are categorized in one of these three categories: diode clamped inverter, flying capacitor inverter and cascaded H-bridge inverter [3]. Regardless of the topology used, the output of these inverters is mainly dependent upon the modulation scheme used. The commonly used modulation schemes are sinusoidal PWM (SPWM) techniques, spaces vector modulation (SVM) and selective harmonic elimination techniques (SHE) [6]. When it comes to implementation of any kind of modulation schemes, a microcomputer such as a simple microcontroller or sophisticated DSP is used to generate switching pulses for the inverter switches. The computational power required from the microcomputer, to generate the switching pulses, will be more when the complexity of the modulation scheme is more. This may be a disadvantageous feature in certain applications where the microcomputer’s processing ability will be required more for other critical system tasks. A trade-off between the modulation schemes is usually made under these situations. Fundamental switching frequency SHE schemes are best suitable for situations where the switching angle generation algorithms need to be comparatively simple.

In all the SHE techniques, the switching angles are found after forming transcendental equations which describe a particular harmonic and solving them. There are several methods [6] available for solving the transcendental equations formed through selective harmonic elimination technique like Newton-Raphson method, graphical analysis method, theory of resultants and optimization algorithms. Newton-Raphson is one of the generally used techniques. But sometimes Newton–Raphson technique suffers from the drawbacks that it converges to local minima. Also judicious choice of initial value is needed to ensure fast convergence. Resultant theory is based on multivariate resultant and involves complex algorithm. So the computation time is high. Optimization algorithms like Genetic Algorithm, Ant colony are used to solve harmonic elimination equations, but their complex nature prevents them from readily using them. On the other hand, graphical analysis method is comparatively simple and straight forward to use.

In this reported work, a five level inverter is controlled by switching pulses whose angles are determined by using fundamental switching frequency selective harmonic elimination technique. The transcendental equations used for obtaining the switching angles are solved by using graphical analysis method. The theory behind graphical analysis method and the process involved in obtaining the solutions of harmonic equations are discussed in Section II. Simulation is performed and the simulated results are presented in Section III.

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The conclusions derived from the obtained results are discussed in Section IV.

II. GRAPHICAL ANALYSIS METHOD FOR SELECTIVE HARMONIC ELIMINATION

A five level inverter like the one shown in Fig. (1a) produces an output waveform as shown in Fig. (1b). It has five levels +Va, 0, +Vb. This waveform exhibits half-wave symmetry and quarter wave symmetry.

Applications of Fourier series will eliminate the even harmonics and the dc component. In this case, the stepped waveform of Fig. 1 can be expressed mathematically [4] as,

\[ V_{\text{(at)}} = \sum_{n=1,3,5...}^{\infty} \frac{4V_{\text{dc}}}{\pi} \left( V_1 \cos(n\theta_1) + V_2 \cos(n\theta_2) + \cdots + V_s \cos(n\theta_s) \right) \sin(\omega t) \]  

(1)

where

- \( s \) is the number of dc sources,
- \( n \) is the harmonic components number,
- \( \theta_1, \theta_2, \theta_s \) refers to the switching angles of each dc level,
- \( V_1, V_2, \ldots, V_3 \) are the multiplication factors used to represent the voltage magnitude of each dc sources.

(for a multilevel inverter with equal dc sources, \( V_1 = V_2 = \ldots = V_s = 1 \)).

The switching angles are so selected to set the fundamental component of the output to the required value and the other harmonic components to zero. Also, the switching angles are selected such that \( \theta_1 < \theta_2 < \cdots < \theta_s \leq \pi/2 \). In order to determine the magnitude of particular harmonic component, \( 'n' \) is substituted with that value in (1).

To demonstrate the application and effectiveness of graphical analysis method in eliminating particular harmonic, a five level cascaded inverter with equal dc sources is chosen and 5th harmonic component has been targeted for elimination. For this case, solution needs to be found for the following transcendental equations:

\[ \cos(\theta_1) + \cos(\theta_2) = \frac{V_1}{4V_{\text{dc}}} \]  

(2)

\[ \sin(5\theta_1) + \cos(5\theta_2) = 0 \]  

(3)

where

- \( V_t \) is the fundamental voltage,
- \( V_{\text{DC}} \) is the dc sources voltage, and
- \( \frac{V_t}{4\pi V_{\text{DC}}} \) is the modulation index '\( m_a ' \).

The solution for equations (2) and (3) are found by using graphical analysis method with the values of \( \theta_1 \) and \( \theta_2 \) lying in the region of interest as shown in Fig(2). Mathematically this relation is expressed as,

\[ \theta_1 < \theta_2 \leq \pi/2 \]  

(4)

As the first step, (2) is used for plotting the relationship between \( \theta_1 \) and \( \theta_2 \). For a particular value of modulation index \( m_a \), by varying the value of \( \theta_1 \), the value of \( \theta_2 \) is found using (2). The values of \( \theta_1 \) and \( \theta_2 \) which doesn’t satisfy the relation mentioned in (4) are discarded. A plot of \( \theta_1 \) and \( \theta_2 \) is drawn as shown in Fig(3).

As the second step, (3) is used for plotting the relationship between \( \theta_1 \) and \( \theta_2 \) using the same procedure as before. The solution of the trigonometric equation of the form

\[ \cos(x) = \cos(y) \]

may have more than one solutions as

\[ x = y, (360 - y), (360 + y), (720 - y), (720 + y) \ldots \text{ so on} \]

Similarly, while solving (3), for a certain value of \( \theta_1 \), multiple values of \( \theta_2 \) may be obtained. Only those values which satisfy (4) are retained and others are discarded. For example, substituting \( \theta_1 = 24 \), \( \theta_2 \) can result in the following values: 12, 60, 84, 132,..... The values 60 and 84 are alone retained as they satisfy the relation \( \theta_1 < \theta_2 < \pi/2 \). The values so obtained are used to plot the relation between \( \theta_1 \) and \( \theta_2 \) as shown in Fig(3).

After obtaining all the plots using the above mentioned procedure, the points where the plot of (3) intersects the plot of (2) will represent the switching angles that are to be used to control the output waveform.
multilevel inverter. For multilevel inverters with unequal dc sources, the procedure can be repeated after modifying the (2) as,

\[ V1 \cos(\theta_1) + V2 \cos(\theta_2) = m_a \]

where \( V2 = Y^* V1 \). In this reported work, \( V2 \) is assumed to be 0.9 times \( V1 \) and the plot shown in Fig.(4) is obtained.

![Figure 4. Plot of \( \theta_1 \) Vs \( \theta_2 \) (for non-equal dc sources)](image)

### III. SIMULATION RESULTS

A single phase, five level inverter has been selected to test the effectiveness of selective harmonic elimination using graphical analysis method. The switching angles deduced using graphical analysis method is used to control the five-level inverter. Simulation has been carried out to check the performance of the inverter. The magnitude of the 5th harmonic component for various values of modulation indexes is tabulated in Table (1). It can be observed from the tabular column that the magnitude of the intended harmonic is either total eliminated or reduced drastically.

<table>
<thead>
<tr>
<th>Modulation index</th>
<th>( \theta_1 ) (degrees)</th>
<th>( \theta_2 ) (degrees)</th>
<th>Magnitude of h5 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>40.3</td>
<td>76.7</td>
<td>0.0</td>
</tr>
<tr>
<td>1.1</td>
<td>33.0</td>
<td>74.6</td>
<td>0.3</td>
</tr>
<tr>
<td>1.2</td>
<td>33.0</td>
<td>69.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.3</td>
<td>29.1</td>
<td>65.1</td>
<td>0.0</td>
</tr>
<tr>
<td>1.4</td>
<td>24.2</td>
<td>60.6</td>
<td>0.9</td>
</tr>
<tr>
<td>1.5</td>
<td>19.8</td>
<td>55.8</td>
<td>0.0</td>
</tr>
<tr>
<td>1.6</td>
<td>14.7</td>
<td>50.7</td>
<td>0.0</td>
</tr>
<tr>
<td>1.7</td>
<td>5.0</td>
<td>45.0</td>
<td>0.5</td>
</tr>
<tr>
<td>1.8</td>
<td>0.6</td>
<td>37.2</td>
<td>0.2</td>
</tr>
<tr>
<td>1.9</td>
<td>14.4</td>
<td>21.9</td>
<td>0.4</td>
</tr>
</tbody>
</table>

In practical situations, without employing additional converters, obtaining equal voltage magnitudes from the dc sources may not be possible. So, non-equal dc sources were also used in the simulation to test the effectiveness of the graphical analysis method. The results obtained are tabulated in Table (4).

### Table 2. Results for inverter with non-equal dc sources (\( V2=0.9V1 \))

<table>
<thead>
<tr>
<th>Modulation index</th>
<th>( \theta_1 ) (degrees)</th>
<th>( \theta_2 ) (degrees)</th>
<th>Magnitude of h5 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>50.5</td>
<td>85.7</td>
<td>1.1</td>
</tr>
<tr>
<td>0.8</td>
<td>47.1</td>
<td>80.0</td>
<td>3.6</td>
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<tr>
<td>1.0</td>
<td>44.1</td>
<td>78.4</td>
<td>0.7</td>
</tr>
<tr>
<td>1.1</td>
<td>22.2</td>
<td>85.4</td>
<td>0.6</td>
</tr>
<tr>
<td>1.2</td>
<td>30.0</td>
<td>75.3</td>
<td>0.1</td>
</tr>
<tr>
<td>1.3</td>
<td>30.0</td>
<td>68.5</td>
<td>0.1</td>
</tr>
<tr>
<td>1.4</td>
<td>22.1</td>
<td>58.6</td>
<td>0.4</td>
</tr>
<tr>
<td>1.5</td>
<td>17.1</td>
<td>53.1</td>
<td>0.1</td>
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<tr>
<td>1.6</td>
<td>11.1</td>
<td>46.2</td>
<td>0.0</td>
</tr>
<tr>
<td>1.7</td>
<td>5.7</td>
<td>38.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The graphical analysis method is observed to be effective for multilevel inverters fed from both equal dc sources and unequal dc sources.

### Conclusions

A five level inverter is controlled by employing graphical analysis method. The solutions for the transcendental equations describing the fundamental and 5th harmonic component are found out by drawing curves for various modulation indexes. The solutions so obtained, are used to generate switching pulses to control the inverter. The simulation results show that the targeted harmonics are eliminated or reduced. It can be concluded from the presented work that a combination of multilevel inverter and graphical analysis method can eliminate the targeted harmonics, resulting in production of better quality waveforms at the inverter output. Also, computational complexity of the microcomputer can be reduced, as the switch control algorithm needs to deal only with the time delay routines for generating switching pulses.

### References


