Implementation and Simulation analysis of low pass FIR filter using different window methods

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Abstract — **Finite impulse response (FIR) filter plays an important role in the processing of digital signal. Designing the FIR filter by MATLAB can simplify the complicated computation in simulation and improve the performance. This paper based on the implementation of low pass FIR (Finite Impulse Response) filter using different window techniques such as rectangular window, hamming window and Kaiser window techniques. MATLAB programming processes are used to characterize the magnitude and phase response of low pass FIR filter and then analyze the input and output signal in frequency and time domain for each window method.**

Index Terms — **FIR, Rectangular Window, Hamming Window, Kaiser Window, FFT, IFFT, MATLAB**

I. INTRODUCTION

Filter is a network used to remove unwanted component of a signal, such as noise. Digital filter better than analog filters because of their better stability, reliability and precision and also do not have matching problem. Communication, image processing, speech processing are the main application area of digital filters. There are two types of digital filters: FIR (Finite Impulse Response), IIR (Infinite Impulse Response). As compare to IIR filter, the FIR filter is a non-recursive (without feedback) structure, finite precision mathematical error is very small, while IIR filter is recursive (with feedback) structure and parasitic oscillation may occur because of feedback mechanism in the operation of IIR filter. FIR filter gives better amplitude and linear phase characteristic and also avoids the drift, noise and distortion as compare to IIR filters. The finite impulse response (FIR) filter is one of the most basic elements in a digital signal processing system, and it can guarantee a strict linear phase frequency characteristic with any kind of amplitude frequency characteristic. FIR used for higher order filter design to meet the design specification.

II. SPECIFICATION OF FIR FILTERS

Transfer function of a linear time invariant filter is expressed as[1]:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0+b_1z^{-1}+---+b_Nz^{-N}}{1+a_1z^{-1}+---+a_Mz^{-M}} \quad (1)$$

Since FIR filter is non-recursive structure it means it has no feedback, Then FIR filter of order N can be expressed as[2] :

$$Y[n] = b_0x(n) + b_1x(n-1) + ---------+b_{N-1}x(n-N+1)$$

$$= \sum_{k=0}^{N-1} b_k x(n-k) \quad (2)$$

Where, b_k = Filter co-efficient.

Transfer function of FIR filter expressed as polynomial of degree N-1 in the variable z^{-1} with impulse response of h(n) expressed as[3]:

$$H(z) = \sum_{k=0}^{N-1} h(k)\, z^{-k} \quad (3)$$

Where h(k) = Impulse response

Since all poles represented at the origin, this characteristic of transfer function shows that FIR filter is stable.

FIR filter gives linear phase characteristic if [4]:

$$h(n) = \pm h(N-1-n) \quad (4)$$

For symmetric condition, frequency response :

$$H(\omega) = H_r(\omega)e^{-j\omega\left(\frac{N-1}{2}\right)} \quad (5)$$

Where, $H_r(\omega)$ = Real function of frequency response and given as :

$$H_r(\omega)=\begin{bmatrix} h\left(\frac{N-1}{2}\right)+2\sum_{n=0}^{\frac{N-n}{2}} h(n)\cos\omega(\frac{N-1}{2}-n) & \text{N odd} \\ 2\sum_{n=0}^{\frac{N}{2}-1} h(n)\cos\omega\left(\frac{N-1}{2}-n\right) & \text{N even}\end{bmatrix} \quad (6)$$

Phase :

$$\varphi(\omega) = \begin{bmatrix} -\omega\left(\frac{N-1}{2}\right) & \text{for } H_r(\omega) > 0 \\ -\omega\left(\frac{N-1}{2}\right)+\pi & \text{for } H_r(\omega) < 0 \end{bmatrix} \quad (7)$$

For antisymmetric condition, frequency response:

$$H(\omega) = H_r(\omega)e^{j\left(-\omega\left(\frac{N-1}{2}\right)+\frac{\pi}{2}\right)} \quad (8)$$

Where, $H_r(\omega)$ = Real function of frequency response and described as :

$$H_r(\omega)=\begin{bmatrix} 2\sum_{n=0}^{\frac{N-n}{2}} h(n)\sin\omega\left(\frac{N-1}{2}-n\right) & \text{N Odd} \\ 2\sum_{n=0}^{\frac{N}{2}-1} h(n)\sin\omega(\frac{N-1}{2}-n) & \text{N Even}\end{bmatrix} \quad (9)$$

Phase :

$$\varphi(\omega) = \begin{bmatrix} \frac{\pi}{2}-\omega\left(\frac{N-1}{2}\right) & \text{for } H_r(\omega) > 0 \\ \frac{3\pi}{2}-\omega\left(\frac{N-1}{2}\right) & \text{for} H_r(\omega) < 0 \end{bmatrix} \quad (10)$$

From above specification of FIR and IIR it is clear that FIR have more advantage as compare to IIR.

III. DESIGN OF FIR FILTER

Design of FIR filter based on different method that described as :

- Fourier series method
- Frequency sampling method
- Window technique

Fourier series method has a disadvantage of oscillation that produces Gibbs Phenomenon.
GIBB'S PHENOMENON: Desired frequency response Hd(ω) obtained with infinite duration unit sample response hd(n). To make a finite duration FIR filter, truncate the hd(n) with n= ± (N-1)/2. But oscillation occurs due to the slow convergence of Fourier series. This is known as 'Gibbs Phenomenon'.
Frequency sampling method can be used for non prototype irregular shape of magnitude response. Disadvantage of frequency sampling method is that only at sampled point, desired frequency response equal to the input frequency response that is obtained by interpolation and it provides error at that point. where it was not being sampled.
Window technique better than other method because it is easy to use, simple, suitable and easily understandable and also it is free from oscillation and ringing effect

A. Procedure To Design Fir Filter By Using Window Method :

First define the edge frequencies of passband and stopband also provide a sampling frequency.
Calculate the order of filter by using ripples and cutoff frequencies of passband and stopband. To design a low pass FIR filter prefer following steps :
Let, ideal frequency response of low pass FIR filter[5][4]:

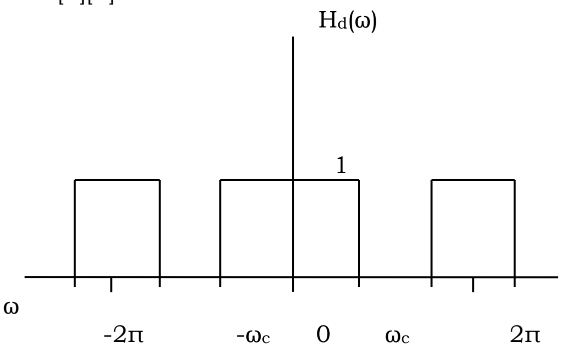

Fig. 1: Frequency response of ideal low pass filter

$$H_d(\omega) = \begin{pmatrix} 1 & 0 \le |\omega| \le \omega_c \\ 0 & \omega_c \le |\omega| \le \pi \end{pmatrix} \quad (11)$$

Filter co-efficient of FIR filter can be calculate by the inverse fourier transform of ideal frequency response $H_d(e^{j\omega})$[1]:

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n} d\omega \quad (12)$$

$$h_d(n) = \frac{1}{2\pi}\left[\int_0^{\omega_c} 1 * e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} 0 * e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1.\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \begin{bmatrix} \frac{2f_c\ \sin(n\omega_c)}{n\omega_c} & n \ne 0 \\ 2f_c & n = 0 \end{bmatrix} \quad (13)$$

Calculate the impulse response co-efficient of FIR filter as[1] :

$$h_d(n) = h(n).w(n) \quad (14)$$

Where, w(n) = window co-efficient (i.e. different for different window technique).
After that draw the magnitude and phase response of FIR filter.

IV. WINDOW TECHNIQUE

In this paper three window techniques are used to design a low pass FIR filter:
i)Rectangular window. ii)Hamming window.
iii)Kaiser window.

Rectangular window:

Weighing co-efficient of rectangular window[2]:

$$W[n] = \begin{bmatrix} 1 & n \le \frac{N-1}{2} \\ 0 & \text{Otherwise} \end{bmatrix} \quad (15)$$

This window technique has not frequently use because of less stopband attenuation and also gives more ripples.

Hamming window:

Hamming window represented by the weighing function[2]:

$$W[n] = \begin{bmatrix} 0.54 - 0.46\cos\frac{2\pi n}{N-1} & 0 \le n \le N-1 \\ 0 & \text{Otherwise} \end{bmatrix} \quad (16)$$

Hamming window have an advantage over rectangular window of less ripple and large width of first side lobe but it have a demerit of high width of transition region.

Kaiser window:

This is described by the weighing function[4]:

$$W[n] = \begin{bmatrix} \frac{I_0(\beta)\left[\sqrt{\left(\frac{N-1}{2}\right)^2 - \left(n - \frac{N-1}{2}\right)^2}\right]}{I_0\left[\beta\left(\frac{N-1}{2}\right)\right]} & 0 < n \le \frac{N-1}{2} \\ 0 & \text{Otherwise} \end{bmatrix} \quad (17)$$

where $I_0(\beta)$ = Bessel's function of first kind
Parameter β expressed as[2] :

$$\beta = \alpha\left[1 - \left(\frac{2n}{N-1}\right)^2\right]^{0.5} \quad (18)$$

β defines the shape of window and also control the tradeoff between the amplitude of side lobes and width of main lobe. Large value of β reduce the window side lobe and therefore ripple also reduce in passband and stopband region.
There are two reasons that make Kaiser window better than others :
Window function with minimum stopband attenuation (A_s= -20log$_{10}$ δ_s) has minimum main lobe width of filter with parameter β as[7]:

$$\beta = \begin{bmatrix} 0.1102(A_s - 8.7) & A_s > 50 \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21) & 21 \le A_s \le 50 \\ 0 & A_s \le 50 \end{bmatrix} \quad (19)$$

Second is length of filter(N) reduce the main lobe width and achieve desired transition region ($\Delta\omega$) and stopband attenuation (A_s) as[7] :

$$N = \frac{(A_s - 7.95)}{14.6(\Delta\omega)} \quad (20)$$

V. ASSIGN PARAMETERS TO DESIGN FIR LOW PASS FILTER

In this paper some specified parameters given as :
Passband ripple = 0.03
Stopband ripple = 0.01
Passband frequency = 450 hz
Stopband frequency = 580 hz
Sampling frequency = 1.5 khz
Filter order =177

These parameters used to getting frequency-gain characteristic and frequency-phase characteristic. Let two frequencies f1=450hz,f2=650hz are used as mixed input sinusoidal signal as[3] :

$$S(t) = \sin(2*\pi*f_1*t)+\sin(2*\pi*f_2*t) \quad (21)$$
$$S(t) = \sin(2*\pi*450*t) + \sin(2*\pi*650*t).$$

This sinusoidal input signal used to analyse the time and frequency response in before filtering and after filtering region.

A. Simulation Result of Low Pass Filter

RECTANGULAR WINDOW

From fig.3 of before filtering response it is clear that four represented frequencies with amplitude of 75db are used for filtering process. Fig.2 shows that range of pass-band lies from 0-600hz and rest is stop-band range upto 1000hz.
After filtering process of fig-4 describe that pass-band frequencies i.e. 450hz and 1050hz kept with amplitude of 23db and frequencies that lies in stopband range are being filtered. In passband region linear phase response is available. This shows best part of low pass filtering

HAMMING WINDOW

From fig-5 it is clear that passband frequency range is 0-600hz and 610-1000hz is stopband frequency range. In fig-6 clearly mention that four frequencies 450hz, 650hz, 850hz and 1050hz with amplitude of 75db are used for filtering process.

After filtering figure shows that frequency that lies in passband frequency range i.e. 450hz and 1050hz frequencies are kept with amplitude of 20db and frequencies that belongs to stopband frequency range are filtered i.e. represented in figure-7. Phase is linear in passband range and don't take attention at stopband phase response. These situations describe low pass filter in best way

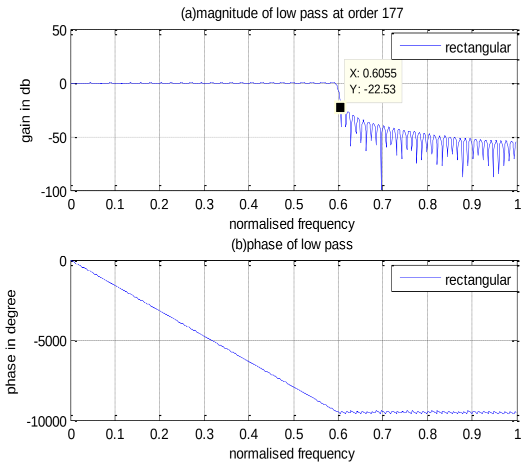

Figure.2 : Magnitude and phase response of rectangular window

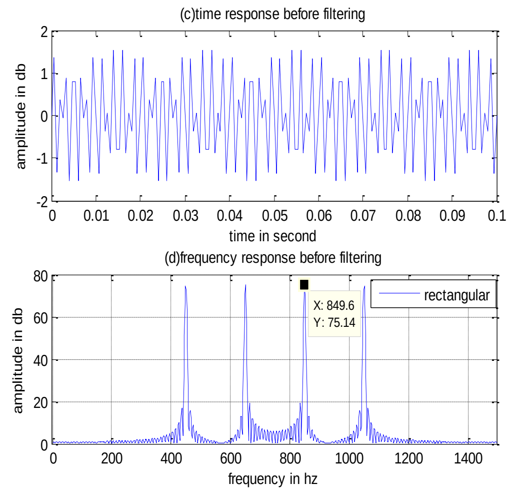

Figure.3 : Before filtering time and frequency response of rectangular window

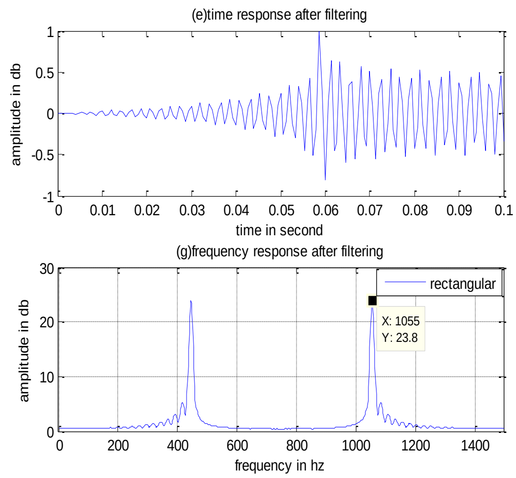

Figure. 4: After filtering time and frequency response of rectangular window

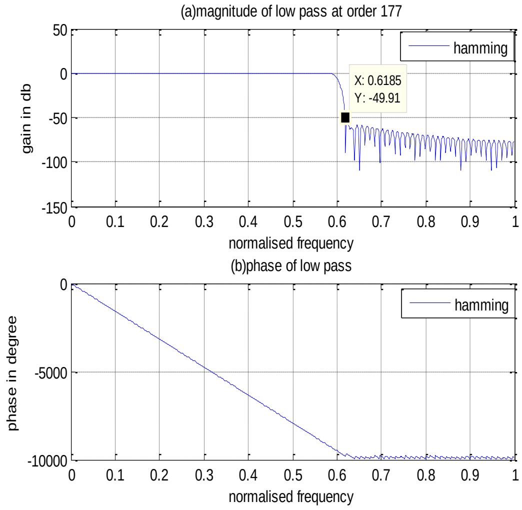

Figure.5: Magnitude and phase response of hamming window

KAISER WINDOW

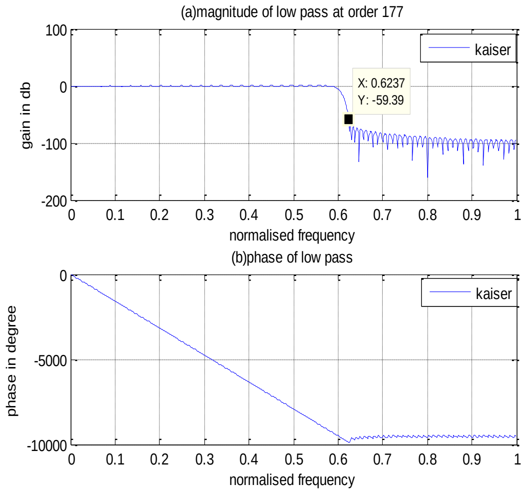

Fig.-6: Magnitude and phase response of Kaiser Window (β = 6.8)

Magnitude response(Fig.-8) of Kaiser Window shows that pass band exist from 0-600Hz and stopband lies from 620-1000Hz. Phase response is linear in pass band frequency.

VI. APPLICATION

Simulation result of low pass FIR filter shows that this filter is used to reduce the noise and frequency boosting problem. After filtering process have an advantage in a field of speech filtering. This design of low pass filtering used to discard the high frequency spectrum of speech signal and get a desired response. Thus this filtering analysis of frequencies plays an important role in speech processing

CONCLUSION

This paper shows the design of low pass FIR filter in rectangular, hamming and Kaiser Window. Hamming window provides less ripple and phase is more linear than rectangular and Kaiser Window. So hamming window is consider as stable window. But Kaiser window method always preferred for designing of FIR filter because of its flexible property.

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REFERENCES

[1] S. Salivahanan, A. Vallavaraj, C. Gnanapriya, “Digital Signal Processing”, Second Edition, Tata McGraw Hill Education Private Limited,2010.

[2] Saurabh Singh Rajput, Dr. S. S. Bhadauria, “Comparison of Bandstop FIR Filter Using Modified Hamming Window and Other Window Functions and Its Applications in Filtering a Multitone Signal”, International Journal of Advanced Research in Computer Engineering and Technology, Volume 1, Issue 8, October 2012.

[3] John G. Proakis, Dimitris G. Manolakis, “Digital Signal Processing Principles, Algorithms and Applications”, Fourth Edition, PHI Publication, 2006.

[4] Apurva Singh Chauhan, Vipul Soni, “Design of FIR Filter on FPGAs using IP Cores”, International Journal of Advancements in Technology,Vol 4, No. 1(March 2013)©IJoAT.

[5] Mbachu C.B., Onoh G.N., Idigo V.E., Ifeagwu E.N., Nnebe S.U., “Processing ECG Signal with Kaiser Window-Based FIR Digital Filters”, International Journal of Engineering Science and Technology, Vol. 3, No. 8, August 2011.

[6] Nirav Raghajibhai Patel, “Design and Performance Comparison of FIR Low Pass Filter using Kaiser Window and Remez Exchange Algorithm”, International Journal of Emerging Trends in Electrical and Electronics(IJETEE- ISSN:2320-9569), Vol. 5, Issue 1, July 2013.

[7] D. Swetha, K. Suneetha Krishna, A. Saritha, V. Shamily, “Design of FIR BSF Using Modified Hamming Window”, Journal of Information, Knowledge and Research in Electronics and Communication Engineering, ISSN:0975-6779|NOV 11 To Oct 12| Volume-02, ISSUE-01.

[8] Lalit Maurya, Vijay Kr. Srivastava, Rajesh Mehra, “Simulink Based Design Simulation Of Band Pass FIR Filter”, IJERT:International Journal of Research in Engineering and Technology, eISSN: 2319-1163|pISSN:2321-7308, Volume:03 Issue :02|Feb 2014.

[9] Rohit Patel. Mukesh Kumar, A.K. Jaiswal, Rohini Saxena, “Design Technique of Bandpass FIR filter Using Various Window Function”, IOSR Journal of Electronics and Communication Engineering(IOSR-JECE), e-ISSN :2278-2834,p-ISSN:2278-8735,Volume 6, Issue 6 (Jul-Aug. 2013), PP 52-57.

[10] Suhaib Ahmed, “Design Analysis of High Pass FIR filter Using Hanning, Bartlett and Kaiser Windows”,International Journal of Advanced Research in Computer Science and Software Engineering, Volume 2, Issue11, November 2012.

[11] Dr. Ziyaad Hussein Saleh, “Design of FIR Filter Based on Adaptive Window”, Modern Applied Science, Vol. 5, No. 3, June 2011.